Applications of Waiting Lines Models for Improving the Performance Levels of Banking Service System

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Abstract- In this study, the theory of waiting queues was applied to improve the quality of banking services provided by Jumhouria Bank, Ahmed Al-Sharif Branch. By using a single-row waiting queue model and several service centers with one service stage (MM/s), and to determine the requirements for building the model, the study focused on banking service centers, where the research relied on studying two periods (the first period is first half of the month and the second period is second half of the month), according to the bank's system, which usually adopts five service centers in the first half of the month and three service centers in the second half of the month. The importance of the study lies in that it addresses the problem of the unwanted waiting phenomenon that most industrial and service institutions suffer from. The aim of study is to develop a model that optimizes the waiting line quantitatively

The service rate and arrival rate were calculated and queuing equations were applied using Quantitative Methods (QM) software (Waiting Lines). The study concluded that the system operates under a stable state, where the value of the utilization rate, which is the probability that the system will be busy in the first period, is 79%, the probability that the system will be empty is 1%, the probability that the system will be busy in the second period is 62%, and the probability is 13% that the system is empty in the second period.

Keywords- Waiting Queues, Arrival Rate, Service Rate, Utilization Rate.

ملخص- تم في هذه الدراسة تطبيق نظرية طوابير الانتظار لتحسين جودة الخدمات المصرفية التي يقدمها مصرف الجمهورية فرع أحمد الشريف. باستخدام نموذج طابور الانتظار ذو الصف الواحد وعدة مراكز خدمة بمرحلة خدمة واحدة(MM/s) ، ولتحديد متطلبات بناء النموذج، ركزت الدراسة على مراكز الخدمة المصرفية، حيث اعتمد البحث على دراسة فترتين (الفترة الأولى هي النصف الأول من الشهر والفترة الثانية هي النصف الثاني من الشهر)، وذلك حسب نظام البنك الذي يعتمد عادة خمسة مراكز خدمة في النصف الأول من الشهر وثلاثة مراكز خدمة في النصف الثاني من الشهر. وتكمن أهمية الدراسة في أنها تعالج مشكلة ظاهرة الانتظار غير المرغوب فيه التي تعاني منها معظم المؤسسات الصناعية والخدمية. وتهدف الدراسة إلى تطوير نموذج يعمل على تحسين خط الانتظار، من خلال النموذج الكمي.

تم حساب معدل الخدمة ومعدل الوصول، وتم تطبيق معادلات الانتظار باستخدام برنامج الأساليب الكمية - طوابير الانتظار (QM). وخلصت الدراسة إلى أن النظام يعمل في ظل حالة مستقرة؛ حيث تبلغ قيمة معدل الاستخدام، وهو احتمال أن يكون النظام مشغولاً في الفترة الأولى 79%، واحتمال أن يكون النظام فارغاً هو 1%. وأن احتمال أن يكون النظام مشغولاً في الفترة الثانية هو 62%، واحتمال أن يكون النظام فارغاً في الفترة الثانية هو 13%.

I. INTRODUCTION

Queuing theory studies a common situation in human life where a resource is limited and can be used by multiple customers, resulting in delays or denial of service to some customers. This is due to the fact that the desire to understand the objective reasons for such delays and rejections and the possibilities of mitigating their effects, motivates the development of queuing theory. This situation occurs where the total number of customers requiring service exceeds the number of facilities. So, we can define a queue as "A group of customers/ items waiting at some place to receive attention/ service including those receiving the service" [1].

Queuing theory is a form of probability that relates to the study of waiting lines (queues). This is for a system with a fixed flow of units (customers) and a fixed number of serving servers (service centers). The analyst wants to know whether the number of service centers in the system is sufficient to handle the inflow of requests. The goal is to calculate various performance metrics of the system. These include the probability that the service will be immediately available to a new arrival, the average number of units in the queue, in the system, and the corresponding times in the queue and system [2].

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Mathematical models of queuing systems (QS) are widely used to study and improve various technical, physical, economic, industrial, administrative, medical, military and other systems. The objects of study in queuing theory are situations in which there is a particular resource and customers need to obtain that resource. Because the resource is restricted, and customers arrive at random moments, some customers are rejected, or their processing is delayed [1].

Therefore, the challenge that facing the modeling and managing the performance of service that customer received is a randomness of arrival time and service time.

The goal of queuing theory is to strike a balance between two extremes by minimizing service costs and maximizing customer satisfaction. Queues are a daily occurrence in most of the commercial banks due to queue mismanagement. This reflects the system's lack of a low, customer centric service rate in the business philosophy[1].

There are claims that the customer is dissatisfied with the bank queuing system in Libya. Hence, the need to ensure the bank's service performance and determine the optimal service strategy for optimal service at the Jumhouria Bank, Ahmed Al-Sharif Branch.

Literature review:

The previous studies, provides a review of empirical studies on the effect of waiting on customer satisfaction and its implications for the managing queues in service delivery. This review will help to identify the relevant variables for the analysis.

At Fidelity Bank Plc, Maiduguri, Bakari C. et al. (2014) used a model M/M/1, and examined the queuing process (at automatic teller machine-ATM), found, the utilization rate lees than one (i.e., $\rho<1$), and concluded the that the system will over utilize when arrival rate is greater than the service rate [3].

A study in Adis- Ababa bank tried to model the bank servers as a queue structure system with several servers, found that the arrival rate follows a Poisson probability distribution, while the service rate of the servers follows an exponential probability distribution. and the number of servers were found Five with a low server utilization rate of 0.578[4].

In order to consider the optimal number of servers in the system two types of costs should be considered, which are, service cost and waiting time cost, such costs can be derived the total operating system cost (service cost and waiting time cost), functionally implies that, the waiting time cost decreases with increases of service rate.[5] Cowdrey G., et al., (2018) applied different queuing strategies using waiting time as a measure of performance, to find out the most efficient solution, the following queuing methods were considered: First-in-First-out (FIFO), Last-in-First-out (LIFO), Shortest Job First (SJF), Longest Job First (LJF. The results showed that the SJF method has the shortest waiting time and the highest customer satisfaction [6].

Previous studies show different models that analyzed and simulated the waiting lines, and come to result that, there are two variables lead the model of waiting lines, service rate, and arrival rate. On the other hand, the waiting time in the queue affected by service rate. Bank management has to balance between service cost and customer satisfaction as a fact.

Practical aspect:

In order to transform the theoretical aspect into a practical model, queuing models were applied to improve the quality of services in the banking services centers at Jumhouria Bank, Ahmed Al-Sharif branch, which suffer from crowding, especially at the beginning of the month.

Study objectives:

The aim of study is to developing a model that optimizes the waiting line, through quantitative model, in order to Improving the quality of service provided by employees using mathematical methods, improve the system performance and customer satisfaction and reaching results that can help banking management in making the appropriate decision to improve its services provided to the community.

Study hypotheses:

The arrival rate follows a Poisson distribution.

The service rate follows an exponential distribution.

The queuing model for the service system in banks is a queuing model with several service channels at one service stage.

The service system followed in the bank is first come, first served.

Methodology:

In this study, the role of waiting lines was determined using a one-row waiting line model and several service centers with one service stage (MM/s) to improve the quality of services provided by Ahmed Al-Sharif Bank. By using the appropriate model and determining the requirements for building the model, the study focused on banking service centers and adopted five service centers in the first period (first half of the month) and three service centers in the second period (second half of the month) as is done in the bank. The service rate and arrival rate were calculated and queuing equations were applied. The study is based on the quantitative aspect using a queuing model with several service channels at one service stage through the Quantitative Methods (QM) program. A single queue model with several service channels as shown in figure (1) which is followed in the Jumhouria Bank, Ahmed Al-Sharif Branch.

II. GENERAL STRUCTURES

Characteristics of queuing system:

The queuing system refers to the number of ranges single, multiple or precedence ranges and their lengths. The type of line depends on the layout of service medium and the length (or size) of a line depends upon functional situations similar as physical space, legal restrictions, and

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station of the guests. In certain cases, a service system is unfit to accommodate further than the needed number of guests at a time. No further guests are allowed to enter until further space is made available to accommodate new guests. Similar type of situations is appertained to as finite (or limited) source line. Exemplifications of finite source ranges are bank, cinema halls, cuffs, etc. On the other hand, if a service system suitable to accommodate any number of guests at a time, also it's appertained to as horizonless (or unlimited) source line.

On arriving at a service system, if guests find long line(s) in front of a service installation, also they frequently don't enter the service system in spite fresh staying space is available. The line length in similar cases depends upon the station of the guests. For illustration, when an automobilist finds that there are numerous vehicles staying at the petrol station, in utmost of the cases, he doesn't stop at this station and seeks service away [7].

Therefore, the mean features can be summarized in to following:

1.Population source.
2.Number of servers (channels).
3.Arrival and service model.
4.Queuing discipline (service order).

Queuing parameters:

The most important information required to solve a waiting line problem is the nature and probability distribution of arrivals and service pattern. The answer to any waiting line problem depending on finding [8].

a) Queue length:

The probability distribution of queue length or the number of persons in the system at any point of time. Further we can estimate the probability that there is no queue.

b) Waiting time:

This is probability distribution of waiting time of customers in the queue. It is necessary to determine the duration a customer spends in the queue before their service begins, known as their queue waiting time. The total time spent in the system is the waiting time in the queue plus the service time. The waiting time depends on various factors, such as:

1. The number of units already waiting in the system.
2. The number of service stations in the system.
3. The schedule in which units are selected for service.
4. The nature and magnitude of service being given to the element being served.

c) Service time:

It is the time taken for serving a particular arrival.

d) Average idle time or Busy time distribution:

The average time for which the system remains idle.

Advantages of queuing theories:

There are Advantages that can be observed or experienced when implementing any of these theories [9]:

1. Minimize unproductive time.
2. Enhance efficient access for individuals seeking services at service delivery centers.
3. Reduce unnecessary expenses.
4. Improve the overall effectiveness of service performance.

Components of the queuing system

The queuing system has basic components that can be identified as follows [10]:

a) Arrival rate:

It is the average number of customers who arrive at service delivery centers to obtain the service they desire during a specific period of time and It is Symbolized by (λ).

b) The length of the queue:

It represents the quantity of customers waiting for service, with waiting queues being subject to the capacity of the service centers. When the service centers have limited capacity, access to them is controlled and predetermined. Conversely, access may come from an unlimited number of service requesters in different scenarios. Regardless, there must always be one or more service centers available.

c) Service rate:

It indicates the average number of units requesting the service that are served during certain periods of time and is symbolized by the symbol (μ).

Service system:

The service system is responsible for organizing customers in the waiting line or service line, as it determines the criteria for selecting customers to be served. There are many ways available for selecting customers from the waiting line and one of the most widely used and fairest methods is the first-come-first-first-served system [11].

Service centers are the centers that provide the service requested by customers. As shown in the following figures, there are four basic channels for waiting lines that determine the format of service centers, which are as follows [9]:

a) Single queue model at single service center:

The service channel (or servers), commonly known as service channels for all customers to pass through one service provider simultaneously over the entire process. Figure (1) shows the structure of this model:

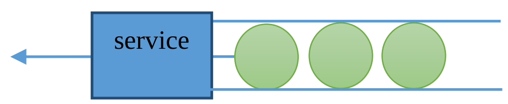

Fig 1. Single queue, single service [9]

b) Single queue model with multiple service stages:

An example of this is production lines with staged industries, where the product passes through many production lines to reach the final form, as figure (2) shows the structure of this model:

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Fig. 2. Single queue, multiple service [9]

c) Single queue model with several service channels:

The parallel arrangement consists of a number of service facilities in parallel so that a customer may join the queue of choice in front of service facilities or may be served by any service facility, as shown in figure (3).

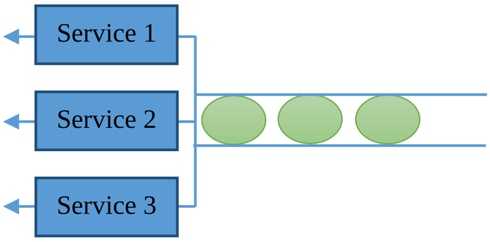

Fig. 3. Single queue, single service [9]

d) Single queue model with several service channels and multiple stages:

An example of this is the registration procedures in universities, as shown in figure (4):

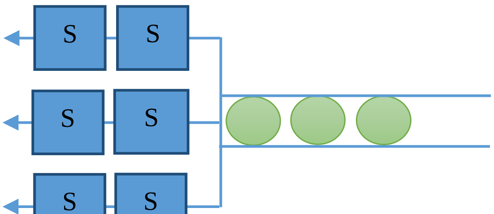

Fig. 4. Single queue, several service and multiple stages [9]

The most important probability distributions

The probability distributions in the theory of waiting queues are, customer arrivals often follow the Poisson theoretical distribution, while service periods follow the exponential distribution. However, this does not negate the existence of other theoretical distributions that both access and service periods can follow.

a) Poisson distribution:

It is called the law of small probability, and it is used in many random processes whose components are generated in a specific unit of time or space. It can be said that customers' arrival to service centers follows a Poisson distribution if Poisson contexts are available, which are:

- The probability of an event Δt occurring in the period depends only on the length of the period. It can be expressed as the constant of the arithmetic mean of the number of accidents in a unit of time, that is, the probability of moving from state n to n-1 is equal, where $\lambda = \lambda n$ is the number of accidents in a certain period independent of accidents in previous periods.
- The probability of two events occurring in the same period is very small. Only one event can occur during the period. The general formula of Poisson's law as follows:

$$P_n(t) = \frac{(\lambda t)^n}{n!} e^{-\lambda}$$

- Any process that follows a Poisson distribution is considered a statistically independent process.
- Any process that follows a Poisson distribution has stable frequencies, meaning that the distribution of events depends on the length of the time period and not on the location of the period.

b) Exponential distribution:

It is used to analyze the number of customers arriving in a certain period of time, as well as the times separating two successive arrivals. It is also used in studying service times and is defined by the following formula:

$$P_n(t) = \mu e^{-\mu t}$$

Where: μ is the service performance rate and λ is the customer arrival rate.

Mathematical Models:

Single-Channel Queuing (M/M/1) it is means that the arrival and service time are exponentially distributed (Poisson process). the following variables will be investigated:

- Arrival rate less than service rate ($\lambda < \mu$).
- The arrival rate (λ) is subject to the Poisson distribution.
- The service rate (μ) is subject to the exponential distribution.
- The approved service system is first come, first served (FCFS).
- The capacity of the system and service requesters is infinite (∞).

In order to understand this type of model, the following mathematical and probabilistic relationships need to be clarified [12]:

μ: Service rate &λ: Arrival rate

- Utilization rate: $\rho = P = \frac{\lambda}{\mu}$
- The probability of the presence of units in the system (the system is working):

$$P = \frac{\lambda}{s \cdot \mu}$$

- Probability of zero customers:

$$P_0 = 1 - \rho$$

- The probability of having n customers:

$$P_n = P_0 \rho^n = (1-\rho)\, \rho^n$$

- The average number of customers in the queue:

$$L_q = \frac{\lambda^2}{\mu(\mu-\lambda)} = \frac{\rho^2}{(1-\rho)}$$

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-The average number of customers:

$$L_s = \frac{\lambda}{\mu-\lambda} = \frac{\rho}{1-\rho} = L_q + \frac{\lambda}{\mu}$$

- The average waiting time in the queue:

$$W_q = P.W\,s = \frac{\lambda}{\mu(\mu-\lambda)} = \frac{L_q}{\lambda}$$

- The average time spent, including the waiting time:

$$W_s = \frac{1}{\mu-\lambda} = W_q + \frac{1}{\mu}$$

The bank working system considered in the study:

The study was conducted only on exchange centers and the current accounts department, and observation periods were determined for the purpose of counting the average number of arrivals to service provision centers. It was noted that on some working days the service is provided by 5 server centers during crowding, and on other days only 3, and the two cases were assumed (days of crowding and days of no crowding). The total viewing period was 10 days, divided into two periods: the crowding period, from (04/02/2024) to (15/02/2024), and the4 other period which is no crowding, from (18/02/2024) to (29/02/2024). The viewing times were from 10:00 am to 02:00 pm, divided into 4 viewing periods, each viewing duration was one hour.

III. Results and Discussion

Arrival rate calculation:

Table (1) shows the number of people who arrived each day and for each hour during the viewing periods as follows:

TABLE 1. ARRIVAL RATE OF CUSTOMERS TO THE BANK

First period						
Time / Day	10 - 11	11 - 12	12 – 13	13 - 14	Total	Average
1	213	173	157	145	688	172
2	193	182	140	137	652	163
3	175	160	138	127	600	150
4	163	142	128	119	552	138
5	155	139	121	113	528	132
Second period						
Day	10 - 11	11 - 12	12 – 13	13 - 14	Total	Average
1	101	97	89	77	364	
2	`96	88	91	78	343	85.75
3	85	66	51	43	245	61.25
4	72	57	63	36	228	57
5	55	46	31	28	160	40

The Arrival rate is calculated by Sum the averages obtained and divided by the number of study days every period.

Average arrival rate $(\lambda_1) = \frac{755}{5}$

= 151 customer per hour.

Average arrival rate $(\lambda_2) = \frac{335}{5}$

= 67 customer per hour.

Service rate calculation:

The service time for each servicer (cashier) during the study days was calculated by calculating 20 customers per cashier per day, divided by working hours throughout the specified period. The following tables show the total and average service rates for the cashiers during the observation period.

a) First period for five services:

Table (2) shows the total and rate of service during the first period from (04/02/2024) to (15/02/2024).

TABLE 2. SERVICE RATES- FIRST PERIOD FOR FIVE SERVICES

Day	service1	service2	service3	service4	service5
	Average	Average	Average	Average	Average
1	1.645	1.412	1.079	1.818	1.514
2	1.238	1.568	1.922	1.477	1.545
3	1.483	1.313	1.052	1.680	1.405
4	2.033	1.776	1.415	2.189	1.879
5	1.654	1.497	1.255	1.833	1.581
Sum	8.053	7.566	6.722	8.997	7.924
Aver.	1.611	1.513	1.344	1.799	1.585

The service rate (μ) is calculated by summing the averages and dividing them by the number of study days as follows:

$$\mu_1 = \frac{1.611 + 1.513 + 1.344 + 1.799 + 1.585}{5}$$

$= \frac{7.852}{5}$ = 1.570 minutes per customer.

$\mu_1 = \frac{60}{1.570}$ = 38.217 customer per hour per service.

For five services:

μ_1 = (38)(5) = 190 customer per hour

λ_1 = 151 customer per hour.

This means that the basic condition $(\lambda_1 < \mu_1)$ has been met, since (152 < 190).

The quantitative data collected during the field study period was analyzed using the Quantitative Methods program, and the following equations were used:

- **Results when (S = 5):**

TABLE 3. PARAMETER RATES – FIRST PERIOD

Parameter	Value	Parameter	Value	Value* 60
M/M/s		Average server utilization (ρ)	0.79	
Arrival rate	151	Average number in the queue (L_q)	2.11	
Service rate	38	Average number in the system(L)	6.08	
Number of servers	5	Average time in the queue (W_q)	0.014	0.84
		Average time in the system(W)	0.04	2.42
		Probability (% of time) system is empty (P_0)	0.01	

Where:

ρ = Average server utilization indicates to the busy rate of cashiers (P) = 0.75, meaning that the probability that the service provider busy is 79%.

L_q= 2.11 : The average number of customers in line is approximately 2 customers.

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L = 6.08 : The average number of customers in the system is approximately 6 customers.

W_q = 0.014 of an hour; which is equivalent to 0.84 minutes, or 50.22 seconds, which is the average time a customer spends in line.

W = 0.04 of an hour; which is equivalent to 2.42 minutes 145 seconds, which is the average time a customer spends in the system.

P_0 = 0. 01 : The probability that are the system is empty (1 %).

- **Results when (S from 4 to 8):**

Table (4) shows a comparison of the results of service centers from 4 to 8 service centers:

TABLE 4 COMPARISON THE RESULTS OF SERVICE CENTERS FIRST PERIOD

Parameter \ Services	4	5	6	7	8
Average server utilization	1	0.79	0.66	0.57	0.5
Average number in the queue (Lq)	148.8	2.11	0.55	0.17	0.06
Average number in the system(L)	152.7 8	6.08	4.52	4.15	4.03
Average time in the queue (Wq)	0.99	0.01	0	0	0
Average time in the system (W)	1.01	0.04	0.03	0.03	0.03

From the previous table it can be noted that:

The service provider's busy rate is 100% when there are 4 cashiers and drops to 79% when there are 5 cashiers and 66% when there are 6 cashiers, which is a significant decrease compared to 4 or 5 cashiers, but there is no significant decrease when comparing the busy rate of 6 cashiers with 7 or 8 cashiers.

It is also noted that the average time a person spends in the system decreases significantly when the number of cashiers is from (4) to (5), and there is no significant decrease between 6, 7, and 8 cashiers, as shown in the figure (5).

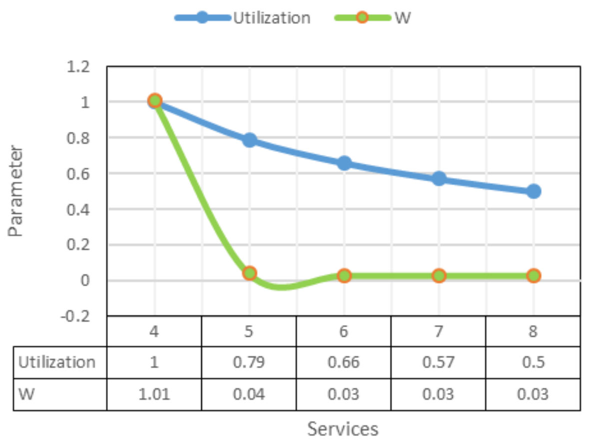

Fig. 5. Comparison between service providers

b) Second period for three services:

Table (3.5) shows the total and rate of service during the second period from 18/02/2024 to 29/02/2024.

The service rate (μ) is calculated by summing the averages and dividing them by the number of study days as follows:

$$\mu_2 = \frac{1.895 + 1.387 + 1.344 + 1.690}{3}$$

$$= \frac{4.972}{3} = 1.657 \text{minutes per customer.}$$

$$\mu_2 = \frac{60}{1.657} = 36.210 \text{ customer per hour per service.}$$

For three cashiers:

μ_2 = (36)(3) = 108 customer per hour

λ_2 = 67 customer per hour.

This means that the basic condition ($\lambda_2 < \mu_2$) has been met, since (67 < 108).

The quantitative data collected during the field study period was analyzed using the Quantitative Methods program, and the following equations were used:

- **Results when (S = 3):**

TABLE 5. PARAMETERRATES - SECANDPERIOD

Parameter	Value	Parameter	Value	Value* 60
M/M/s		Average server utilization	0.62	
Arrival rate (lambda)	67	Average number in the queue (L_q)	0.62	
Service rate(mu)	36	Average number in the system(L)	2.48	
Number of servers	3	Average time in the queue (W_q)	0.01	0.56
		Average time in the system(W)	0.04	2.22
		Probability (% of time) system is empty (P_0)	0.13	

Where:

ρ = The probability that the service provider is busy is 62 %.

L_q = The average number of people in line is approximately 1 customer.

L = The average number of customers in the system (bank) is approximately 3 customers.

W_q = 0.014 of an hour, which is equivalent to 0.84 minutes, or 50.22 seconds, which is the average time a customer spends in line.

W = 0.01 of an hour, which is equivalent to 0.6 minutes 36 seconds, which is the average time a customer spends in the system (the bank).

P_0 = 0. 13 : The probability that are the system is empty (13 %).

- **Results when (S from 2 to 6):**

Table (6) shows a comparison of the results of service centers from 2 to 6 service centers:

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TABLE 6. COMPARISON THE RESULTS OF SERVICE CENTERS-SECAND PERIOD

Parameter \ Services	2	3	4	5	6
Average server utilization	0.93	0.62	0.47	0.37	0.31
Average number in the queue (Lq)	12.02	0.62	0.12	0.03	0.01
Average number in the system (L)	13.88	2.48	1.98	1.89	1.87
Average time in the queue (Wq)	0.18	0.01	0	0	0
Average time in the system (W)	0.21	0.04	0.03	0.03	0.03

From table (6) it can be noted that:

The service provider's busy rate when there are 2 cashiers is 93% and decreases to 62% when there are 3 cashiers and to 47% when there are 4 cashiers, which is a significant decrease compared to 2 or 3 cashiers, and there is no significant decrease after the number of cashiers is 5 and 6. It is also noted that the average time a person spends in the system decreases significantly when the number of cashiers is from (2) to (3), and there is no decrease between 4, 5, and 6 cashiers, as shown in figure (6).

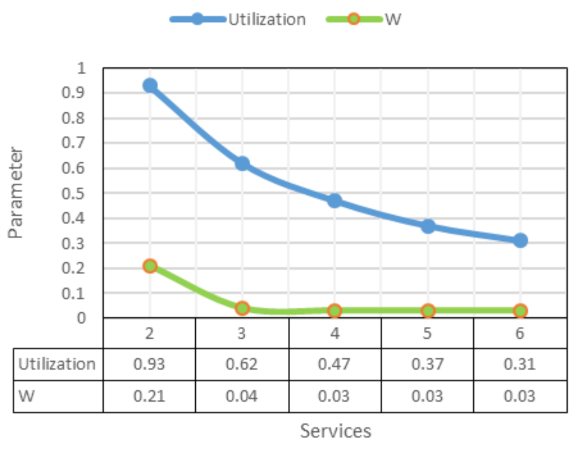

Fig. 6. Comparison between service providers

IV. Conclusions

Through modeling the process, the following conclusions have been derived:

1. It is noted that in the first period there was severe crowding in the bank, which caused the bank to use 5 service centers, and in the second period only 3 service centers were enough due to low crowding,
2. Working with five service centers in the first period was appropriate, as the server utilization rate (probability service provider busy) was 79 % and the average time spent by the customer in the system was approximately 2.4 minutes.
3. If the number of service providers decreases in the first period from 5 to 4, this will lead to an increase in the server utilization rate from 79 % to 100 % and the average time spent by the customer in the system is approximately 61 minutes and this will lead to an increase in waiting and crowding in the bank.
4. If the number of service providers is increased from 5 to 6, this will lead to a decrease in the server utilization rate from 79% to 66%, and the average time the customer spends in the system will be approximately 2 minutes.
5. Increasing the number of service centers more than 5 in the first period leads to a severe decrease in the rate of server usage while the average number of customers in the system and the average time of customers in the system remain constant. This will lead to an increase in the cost because the time spent by the customer in the system whether the number of servers is 5 or 6 is similar.
6. Working with three service centers in the second period was appropriate, as the server utilization rate reached 62%, and the average time spent by the customer in the system was about 2.4 minutes.
7. Increasing the number of service centers more than 3 in the second period leads to a severe decrease in the server utilization rate, while the average number of customers and the average time of customers in the system remain constant. This will result in extra cost, because the customer time spends in the system when the number of the servers is more than 3 is similar.

V. Recommendations

- From analyzing the model, the ideal policy for bank management is 5 service centers in the first period and 3 in the second period. A periodically review is recommended due to any unforeseen changes that may influence on queue parameters.
- In future studies, it is suggested to study the cost estimation associated with the number of service centers in the system.

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