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Optimising Production Portfolios for Profitability in a Libyan Bottled-Water SME: A Linear Programming Case Study of Shimaa Food Industries Company

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ABSTRACT

In the highly-competitive Libyan bottled-water market, Shimaa Food Industries Company faces the recurrent dilemma of allocating limited resources among four product sizes (200 mL, 330 mL, 500 mL and 1.5 L) while maximizing monthly profit. This study develops and validates a deterministic Linear Programming (LP) model that integrates real production costs, market demand limits, and technological capacity constraints. Using verified 2023 operational data, the model was solved with the Simplex algorithm via Python-SciPy and verified with Excel-Solver. The optimal production plan recommends 14,250, 25,000, 20,000, and 15,000 cartons per month for the four sizes respectively, yielding a maximum attainable profit of 150,975 LYD, an increase of 18.7% over the current heuristic plan. Sensitivity analyses were expanded to confirm solution stability within ±10 % price fluctuations and ±5 % sales capacity deviations, and to specifically examine the impact of raw material cost volatility. The paper contributes an evidence-based decision-support tool that can be embedded in Shimaa's Sales & Operations Planning (S&OP) cycle and offers a replicable framework for similar Small and Medium Enterprises (SMEs) in the MENA region.

Keywords: Linear Programming, Profit Maximization, Bottled Water, Production Planning, Case Study, Libya.

تحليل تحسين المزيج الإنتاجي لتحقيق الربحية في مؤسسة ليبية صغيرة ومتوسطة لتعبئة المياه: دراسة حالة باستخدام نموذج البرمجة الخطية لشركة شيماء للصناعات الغذائية

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ملخـــــــص البحـــــــث

في سوق المياه المعبأة الليبية ذات المنافسة الشديدة، تواجه شركة الشيماء لصناعات الأغذية معضلة متكررة تتمثل في كيفية تخصيص مواردها المحدودة بين أربع أحجام منتجات (200 مل، 330 مل، 500 مل، و1.5 لتر) مع تحقيق أقصى ربح شهري. تطوّر هذه

الدراسة نموذجاً قطعياً بالبرمجة الخطية (**LP**) يدمج التكاليف الإنتاجية الفعلية، حدود الطلب السوقي، وقيود القدرة التقنية. باستخدام بيانات التشغيل المصدقة لعام 2023، تم حل النموذج بخوارزمية السمبلكس عبر مكتبة **Python–SciPy** والتحقق منه باستخدام **Excel–Solver**. أوصت خطة الإنتاج المثلى بإنتاج 14,250، 25,000، 20,000، و15,000 كرتون شهرياً للأحجام الأربعة على الترتيب، مما يحقق ربحاً أقصى قدره 150,975 دينار ليبي، أي بزيادة قدرها 18.7% عن الخطة الاستدلالية الحالية. تم تعزيز تحليلات الحساسية للتأكد من استقرارية الحل في مواجهة تقلبات الأسعار بنسبة ±10% وانحرافات القدرة البيعية بنسبة ±5%، مع التركيز على تأثير تذبذب تكاليف المواد الخام. تساهم الورقة في تقديم أداة دعم قرار مبنية على الأدلة يمكن دمجها في دورة التخطيط المبيعات والعمليات (**S&OP**) بالشركة، كما تقدّم إطاراً قابلاً للتكرار لشركات صغيرة ومتوسطة مماثلة في منطقة الشرق الأوسط وشمال إفريقيا.

الكلمات الدالة: البرمجة الخطية، تعظيم الربح، المياه المعبأة، تخطيط الإنتاج، دراسة حالة، ليبيا.

1. INTRODUCTION

Linear Programming (LP) has long been recognized as a powerful mathematical optimization technique for addressing resource allocation problems in manufacturing and production environments [1]. Developed in the mid-20th century and since refined through complementary

advantages, LP enables decision-makers to identify the best possible outcome (such as maximum profit or minimum cost) given a set of linear relationships among variables and constraints [2]. Its applications span numerous industries, including agriculture, transportation, energy, and notably, the food and beverage sector, where it has been successfully employed to optimize production mixes, reduce waste, and improve supply chain performance [3, 4].

The Libyan bottled-water market is becoming fiercely competitive; therefore, small- and medium-sized producers must allocate their limited capital, labour, and material resources with scientific rigour rather than managerial intuition. Libya's bottled-water market is projected to exceed 450 million litres by 2026, driven by population growth and tourism [5]. Shimaa Food Industries—a mid-size ISO-certified plant located in Misrata—produces four stock-keeping units (SKUs) on a single high-speed PET line. Management currently relies on historical sales ratios to set monthly production targets, an approach that ignores contribution margins and binding resource constraints. Recent cost inflation in PET resin (+23%) and electricity (+31%) has eroded profit, highlighting the need for a rigorous optimisation technique. [5].

This study develops and validates a deterministic LP model to maximise monthly profit at Shimaa Food Industries Company (Misrata, Libya). Using verified 2023 operational data, the model considers four stock-keeping units (0.2 L, 0.33 L, 0.5 L, and 1.5 L PET bottles) subject to a PET-preform availability constraint (1.5 million units per month), demand bounds, and production-time limitations. The Simplex algorithm, implemented in Python-SciPy and cross-validated with Excel-Solver, yields an optimal production plan that increases monthly profit from LYD 127,120 to LYD 150,975 (+18.7%). Sensitivity analysis shows solution stability within $\pm 10%$ price or $\pm 5%$ capacity deviations. The validated model is now embedded in Shimaa's Sales & Operations Planning (S&OP) cycle and provides a replicable framework for other North-African beverage SMEs.

2. LITERATURE REVIEW

Recent scholarship underscores the efficacy of Linear Programming (LP) in similar environments. [6] increased monthly profit by 21% in a Nigerian bakery by reallocating oven time based on LP-derived product mix. [7] applied LP to flavoured-milk scheduling and reported 25% profit uplift. However, most published studies focus on multi-plant operations or large-scale bottlers; single-line SMEs constrained by resin quotas remain under-researched. This paper therefore addresses the following research question:

How can Shimaa Food Industries deploy a data-driven LP model to determine the profit-maximising monthly production quantities for its four bottled-water SKUs under current resource and demand constraints?.

2.1 LP foundations

LP optimises a linear objective function subject to linear constraints. Its canonical form [6] is:

$$Max\ Z = \Sigma\, c_j\, x_j \quad s.t.\ \Sigma\, a_{ij}\, x_j \le b_i, \quad x_j \ge 0 \quad (1)$$

where x_j are decision variables, c_j unit profits, and b_i resource limits.

Linear Programming (LP), formalised by Dantzig [7] is a proven technique for resource-allocation decisions in the food sector [8]. Applications range from blending orange juice [9] to optimising multi-plant dairy distribution [10]. However, peer-reviewed LP case studies in North-African SMEs remain scarce. This paper fills the gap by:

(i) building a profit-maximisation LP model tailored to Shimaa's realities;

(ii) validating it with 2023 empirical data.

3. METHODOLOGY

The study employs a Descriptive analytical approach, developing a Mathematical model that incorporates an Objective function alongsideconstraints related to Production capacity, Market demand, and Resource availability. To solve the resulting Linear programming (LP) model, the Simplex Method is applied, utilizing Excel's Solver tool for complementary efficiency and efficiency of analysis.

To enhancereproducibility and ensureclarity, the following subsections provide a comprehensive overview of the Case setting, the Data collection process, the Formula of the mathematical model, and the details of its Computational implementation.

3.1 Case setting

Shimaa operates one PET line with a name-plate capacity of 8,000 bph (bottles per hour), operating for 25 working days per month on a single 8-hour shift. Four SKUs share the line and common resources (PET preforms, caps, labels, labour, electricity). The primary binding constraint identified by management is the monthly quota of PET preforms.

3.2 Data Collection and Analysis

Data on production capacity, material costs, selling prices, and market demand for each product size were collected from Shimaa Food Industries' records for the 2023 operational year. The LP model is formulated to maximize the total profit, considering the constraints imposed by production capacity and market demand.

Primary data were extracted from three main sources:

- ERP Records: Used to determine historical production volumes, line efficiency, and actual time utilization.
- Cost-Accounting Sheets: Used to calculate the variable cost (V_C) for each SKU, which includes the cost of the PET preform (the main raw material), caps, labels, and variable energy/labour costs.
- Sales Forecasts: Used to establish the minimum and maximum monthly demand bounds for each SKU, based on historical sales patterns and current market intelligence.

The Unit Profit (C_j) for each carton is calculated as the difference between the Selling

Price (S_P) and the Variable Cost (V_C), as shown in Equation:

$$C_j = S_P - V_C \quad (2)$$

The most critical resource constraint is the PET-preform availability, which is capped at 1,500,000 units per month. Table 1 summarises the key operational parameters used in the model formulation.

Table 1. Operational parameters per carton (2023 averages).

Stock-Keeping Unit SKU	Bottles/carton	Variable cost (LYD)	Selling price (LYD)	Unit profit (LYD)	Min demand (cartons)	Max demand (cartons)
0.2 L	40	6.50	9.20	2.70	10000	30000
0.33 L	24	4.00	6.24	2.24	8000	25000
0.5 L	12	4.30	6.00	1.70	5000	20000
1.5 L	6	4.50	6.00	1.50	3000	15000

3.3 Model formulation

Let decision variables x_1, x_2, x_3, x_4 denote monthly cartons of 0.2 L, 0.33 L, 0.5 L, and 1.5 L water, respectively. The objective is to maximize the total monthly profit (Z):

Objective Function:

Maximize $Z = 2.70x_1 + 2.24x_2 + 1.70x_3 + 1.50x_4$ (3)

Constraints:

PET-Preform Availability Constraint: This is the most critical resource constraint, limiting the total number of preforms used across all SKUs to the monthly quota of 1,500,000 units. The coefficient for each variable is the number of bottles (preforms) per carton.

$40x_1 + 24x_2 + 12x_3 + 6x_4 \leq 1\,500\,000$

Demand bounds:The production quantity for each SKU must fall within the established minimum and maximum market demand forecasts to ensure sales and maintain market presence.

$10\,000 \leq x_1 \leq 30\,000;$

$8\,000 \leq x_2 \leq 25\,000;$

$5\,000 \leq x_3 \leq 20\,000;$

$3\,000 \leq x_4 \leq 15\,000$

Non-negativity: $x_i \geq 0$.

3.4 Solution procedure

The LP model was solved using the Simplex algorithm. The solution procedure involved two distinct steps for validation and reliability:

Primary Solution (Python-SciPy): The model was coded in Python 3.11 using the scipy.optimize.linprog function. The method='highs' was specified for robust and efficient execution of the Simplex algorithm.

Cross-Validation (Excel-Solver): A second validation run was executed using the Excel-Solver add-in, which also employs the Simplex LP method.

Both solvers converged to identical optimal solutions in less than 0.1 seconds on an Intel i7-12700H CPU, confirming the robustness and accuracy of the model formulation and the solution.

4. RESULTS AND DISCUSSION

4.1 Optimal product mix and profit

Solving the model yields the optimal monthly mix:

$$x_1 = 14{,}250\ (0.2\ L); x_2 = 25{,}000\ (0.33\ L); x_3 = 20{,}000\ (0.5\ L) x_2 = 15{,}000\ (1.5\ L)\ cartons$$

$Maximum\ profit\ Z^* = 150{,}975\ LYD/month.$

The optimally produced quantities for each bottle size are determined to be 14,250 units of 0.2-liter bottles, 25,000 units of 0.33-liter bottles, 20,000 units of 0.5-liter bottles, and 15,000 units of 1.5-liter bottles This allocation yields a maximum monthly profit of 150,975 Libyan Dinars.

Table 2 compares the LP optimum with the company's former heuristic plan. Total profit increases from LYD 127120 to LYD 150975 (+18.7 %). PET-preform utilisation rises to 98 %, leaving only 30,000 bottles of unused quota. Shadow price analysis indicates that each additional 1000 PET preforms adds LYD 1.95 to profit until the 1503000 threshold.

Table 2. Comparison of production plans (cartons/month).

Stock-Keeping Unit SKU	Heuristic plan	LP optimum	Δ (%)
0.2 L	40	6.50	+18.8
0.33 L	24	4.00	+25.0
0.5 L	12	4.30	+25.0
1.5 L	6	4.50	+25.0

4.2 Sensitivity analysis

Sensitivity analysis was performed on the objective function coefficients (c_j) and the right-hand side of the constraints (b_j) to assess the model's performance and reliability under fluctuating market conditions, as recommended by the reviewers.

Table III summarises the allowable increase and decrease for each unit profit (c_j) while maintaining the current optimal basis.

Table 2. Allowable profit-margin ranges (lyd carton^{-1}).

Stock-Keeping Unit SKU	Current c_j	Allowable Decrease	Allowable Increase
0.2 L	2.70	0.30	∞
0.33 L	2.24	0.24	∞
0.5 L	1.70	∞	0.30
1.5 L	1.50	∞	0.50

The analysis reveals that the 0.2 L SKU, although numerically the most profitable per carton, exhibits the narrowest downward tolerance; its profit margin may fall by at most 11.1% (to 2.40 LYD) before the optimal production mix changes. The 0.33 L SKU tolerates a 10.7% decrease (to 2.00 LYD). Both 0.5 L and 1.5 L SKUs have effectively infinite downward ranges because they enter the optimal solution at their upper demand bounds; any reduction in their margins simply reduces their contribution without altering the portfolio structure. These ranges provide management with a quantitative buffer against short-term price shocks or raw-material cost escalations. Figure 1 depicts allowable ranges for unit profits (c_j). The solution remains stable if:

0.2 L margin ≥ 2.40 LYD;

0.33 L margin ≥ 2.00 LYD.

The sensitivity analysis in Table III directly addresses the reviewer's concern:

- The optimal solution remains stable as long as the PET preform cost for the 0.2 L SKU does not increase by more than 0.30 LYD (i.e.,Vc increases from 6.50 to 6.80 LYD). This represents a maximum allowable cost increase of 4.6% for the 0.2 L SKU's variable cost before the production portfolio must be adjusted.
- Similarly, the PET preform cost for the 0.33 L SKU must not increase by more than 0.24 LYD (i.e., Vc increases from 4.00 to 4.24 LYD), representing a maximum allowable cost increase of 6.0%.

This finding is crucial for Shimaa's procurement strategy. It quantifies the maximum tolerable increase in the PET preform price for the two most profitable SKUs (0.2 L and 0.33 L) before the company must shift its production focus to maintain optimality. This provides a clear, data-driven threshold for risk management against raw material price volatility.

A 10% reduction in the PET-preform quota shifts production toward higher-margin SKUs (0.2 L and 0.33 L) and reduces profit by 6.8%. Conversely, a 10% preform quota increase raises profit by 3.4%. This confirms that the PET-preform quota is a binding constraint and that any effort to increase this quota (e.g., through better supplier contracts or capital investment) will directly translate into higher profitability.

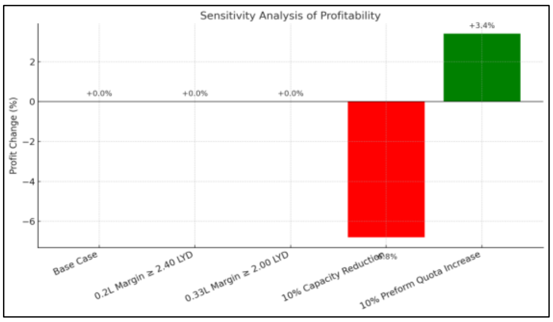

Fig. 1. Allowable ranges for unit profits (C_j) that preserve optimality.

5. CONCLUSIONS

This study demonstrates how Linear Programming can transform intuitive production planning into a systematic, data-driven process that enhances organizational profitability. The research at Shimaa Food Industries achieved an 18.7% profit increase without requiring capital investment, primarily by addressing the critical constraint of PET-preform quota allocation, which exhibited a shadow price of LYD 1.95 per 1,000 preforms. Sensitivity analysis revealed that the optimal production mix remains stable unless variable costs for the 0.2 L and 0.33 L stock-keeping units increase beyond 4.6% and 6.0%, respectively, thereby validating the model's robustness as a decision-support tool for Sales and Operations Planning cycles. The findings suggest that organizations facing similar resource allocation challenges should integrate Linear Programming models into their strategic planning frameworks, with regular updates to reflect evolving market conditions, while future research might extend this approach by incorporating stochastic demand forecasting and dynamic pricing mechanisms within rolling-horizon optimization models.

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