ARTICLE INFO

Article history:
Received 01 May, 2023
Accepted 28 May, 2023
Published 22 July, 2023

Nonlinear Estimation of Incident Solar Irradiance on Finland Using Discrete-time Extended Kalman Smoothers

Mahmoud Elsanhoury[1], Caner Çuhac[2], Janne Koljonen[1], Mohammed Elmusrati[1]

[1]*University of Vaasa*, Vaasa, Finland
[2]*Asyaport Liman A.Ş*, Tekirdağ, Turkey

Firstname.lastname@uwasa.fi[1], ccuhac@asyaport.com[2].

Abstract- A huge source of energy lies within the incident solar heat and light from the sun, especially for colder territories found in the Nordics such as Finland. For a country situated with close proximity to the North pole, Finland suffers from chilling conditions most of the year in addition to the withheld Sun light. Except during the very short seasons of spring and summer when the Sun remains in the sky for almost 22 hours per day on average. Thus, solar heat flux and solar irradiation become essential energy sources between April and September annually. However, solar irradiance is a physical quantity that is highly affected by probabilistic uncertainty found in many weather conditions, solar behavior throughout the year, time of the day, and the geographical location. In this article, we propose a solar irradiance estimation method base on the Extended Kalman algorithms. The experiment was carried out between the years 2014-2016 in which we installed solar radiation sensors underground and on a rooftop inside the campus of Vaasa University. The readings were collected via an embedded system specifically designed for the endeavors of this experiment. The results showed that the algorithm was able to predict the incident solar irradiance on the city of Vaasa (Finland) with an acceptable accuracy range.

Keywords— solar irradiance, extended Kalman filter, probability and stochastics

الملخص: مصادر الطاقة المتجددة صارت ضرورية جدًا وتتزايد أهميتها مع الشح في الحصول على مصادر الطاقة الاحفورية مثل النفط والغاز لأسباب طبيعية مثل محدودية وجودها أو لأسباب سياسية وغيرها .أهمية الطاقة المتجددة مثل الطاقة الشمسية لها أهمية خاصة في بلدان اقصى الشمال الأوروبي مثل دول البلطيق و التي تعاني من البرد الشديد في الشتاء المظلم والذي يمتد لمدة قد تصل الى 6 أشهر في السنة تقريبا. وفي هذه الدول ومنها فنلندا يوجد توجه في إمكانية تخزين الطاقة الحرارية من الشمس لتلبية احتياجات الطاقة في الصيف وإمكانية تخزين هذه الطاقة لاستعمالها خلال فصل الشتاء المظلم. الصيف في فنلندا عكس الشتاء حيث تشرق الشمس لأكثر من 20 ساعة في اليوم في بعض المناطق وبالتالي فإمكانية الاستفادة من النهار الطويل تعتبر من المواضيع البحثية المهمة في فنلندا ودول الشمال بشكل عام .في هذا البحث قمنا باستحداث متحسس حراري لاسلكي يقوم بتجميع معلومات الحرارة من الشمس وتم دفنه تحت الاسفلت في موقف سيارات الجامعة في مدينة فازا وجهاز لقياس كثافة الاشعة الشمسية وتم وضعه على اعلى مبنى الجامعة. الغرض من تجميع البيانات هو الحصول على نموذج دقيق حول العلاقة بين الحرارة التي يمكن تجميعا وكثافة الاشعة الشمسية مع أيام الصيف. المشكلة ان البيانات التي يتم تجميعها وهي ضخمة جدا تكون متأثرة بضجيج مضاف وكذلك بعض الانقطاع أو الحيود في البيانات. من المشاكل مثلا عندما تتوقف سيارة في مكان تجميع البيانات فيسبب منع الاشعة الشمسية أو عبور سحابة او غيوم. أو أحيانا حدوث اعطال في تزويد الطاقة للمتحسس اللاسلكي. وبالتالي نحتاج الى معالجة هذه البيانات وتنقيتها من الضجيج المرافق وكذلك توقع البيانات الناقصة وكذلك الدمج الأمثل لبيانات المتحسس الحراري مع مقياس الاشعة الشمسية. ولهذا فإننا في هذه المقالة قدمنا تقنية بسيطة لتقدير الإشعاع الشمسي الواقع على فاسا باستخدام مرشح كالمان

الموسع (EKF) وتمهيد البيانات (ERTS) للتوقعات. أظهرت النتائج أداءً غير مسبوق للخوارزمية المعطاة التي أفضت إلى تقدير مقبول لدقة النموذج المقترح. بالنسبة للأعمال المستقبلية، نعتزم تطوير نموذج ديناميكي أكثر تقدمًا يشبه الإشعاع الشمسي في فنلندا عن طريق اعتبار عوامل أخرى مساهمة تشمل القيم الفضائية الخارجية وتأثيرات الامتصاص الجوي .

الكلمات المفتاحية: الإشعاع الشمسي، مرشح كالمان الموسع، الاحتمالات والقيم العشوائية

I. INTRODUCTION

The sun provides our planet with renewable sources of energy, not only found in light energy but also in heat energy. The earth is being showered by massive bursts of energy coming from the sun every moment throughout the day. It is estimated that most of the utilized energy from the sun is light-based while the most wasted energy is the renewable heat and warmth incident from our star.

Solar irradiance, also known as “solar insolation”, it is the quantity of sunlight power received from the sun per unit area on earth’s surface, measured in watt per meter square [W/m2]. Nearly 30% of the incident light power received on earth’s surface is attenuated by earth’s atmosphere i.e. the solar irradiance outside the atmosphere (extra-terrestrial) is always greater than the solar irradiance on earth’s surface (terrestrial) [1]. Moreover, knowing the quantity of incident solar irradiance is very beneficial for some photovoltaic scheduling applications [2].

The amount of incident solar irradiance depends on numerous factors and parameters. This physical quantity is highly sensitive to the weather conditions, time of the year and time of the day, the geographical location where it is been measured, and the sensitivity of the measuring equipment. Therefore, the nature of solar irradiance is found to be nonlinear stochastic process that contains higher degrees of uncertainty, which also make the future predictions are very challenging.

The authors of [2] proposed an accurate model to forwardly predict solar irradiation for the next 24 hours based on the analysis of the post-processing of the recorded datasets by adapting the order of the utilized polynomial functions. Furthermore, the authors expanded the proposed method to render a novel method comprised a bank of 24 Kalman filters working simultaneously on modifying the polynomial coefficients to estimate solar irradiance inside an airport. The results showed an acceptable accuracy of root mean square error = 20 W/m2.

The novelty in the article consists of the real datasets being collected from the geographical location (Vaasa, Finland) for 3 successive years, in addition to the lightweight algorithmic methodology that processes the recorded data.

The rest of article is organized as follows: Section I addresses the implemented procedures to collect the solar irradiance on Vaasa, Finland. Section II shows the detailed steps to our methodology in treating the datasets (pre-processing and post-processing) in addition to Kalman algorithms used to obtain most accurate predictions to solar irradiance. Section III describes the developed prediction model to estimate the solar irradiance values in Vaasa. Then, the article concludes with conclusions and references sections.

II. MEASURING SOLAR IRRADIANCE IN FINLAND

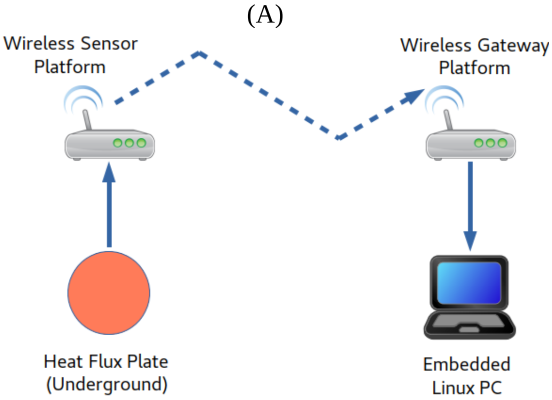
In this study, three independent types of measurements via three sensor devices were pursued: 1) solar irradiance using pyranometer device, 2) heat flux absorbed by asphalt using a heat flux plate buried at depth 5 cm beneath the asphalt layer, 3) the temperature distribution through the depth using a distributed temperature sensing (DTS) system. The three methods are complementary to each other to produce a reliable perspective of the different ground layers. [1]

The data collection site was embedded underneath the University of Vaasa – Palosaari campus between 2014—2016 as illustrated in Figure 1. Data collection site at the University of Vaasa, Finland.. The measured values were transferred from the sensors to the wireless sensor network implemented onsite, thus values were recorded as raw data by the server.

Figure 1. Data collection site at the University of Vaasa, Finland.

Then, the data from the server were received by the wireless gateway, which hand it over to the linked embedded PC for preprocessing using MATLAB, finally it was stored on a hard drive [1]. The wireless sensor network structure is shown in Figure 2.

(A)



(B)

Figure 2. A) The location of pyranometer on the roof top of Tritonia building, Vaasa. B) An illustration depicting the embedded system for data collection which comprises the wireless sensor network.

The conducted experiment to measure the quantity of incident solar irradiance in Vaasa was carried out through the installation of a Hukseflux pyranometer device on the lower rooftop of one of the campus buildings (Tritonia tower) in Vaasa University, which is roughly 11 meters high. The pyranometer location was pointed directly to the open skies without suffering any projected shade from the surroundings. The timestamped samples measured by the device were sent and stored periodically to the server storage via wireless sensor network [1]. Between the years 2014–2016, the pyranometer device had gathered approximately 4.5 million samples of data comprised the measured solar irradiance in W/m^2 (Watts per square meter) every 10 seconds i.e. around 8640 samples per day. The typical amount of solar irradiance for a clear sunny day in Vaasa (latitude 63.102^o) should follow a bell-shaped curve whose peak is around noon time when the sun is at its zenith angle (perpendicular on the surface). The equations that govern calculating solar irradiance are:

$$I_s = I_c \times (\sin\phi \cos\delta + \cos\phi \cos\delta \cos H) \quad (1)$$

$$\delta = 23.45° \times \sin\left(\frac{360}{365} \times (284 - d)\right) \quad (2)$$

$$H = 15° \times (T - 12) \quad (3)$$

where,

I_s irradiance power in W/m^2

I_c global irradiance constant (1000-1376) W/m^2

ϕ latitude angle from which I_s is measured

δ declination angle in degrees

H hourly angle per day time

T time of the day in 24-hour format

d number of days elapsed since 01/01/20xx

Solar irradiance is -naturally- a fluctuating physical quantity associated with a high degree of uncertainty also is directly affected by numerous factors and parameters, such as: weather conditions, time of the year, time of the day, sensor bias, and geographical location. Hence, predicting the amount of incident solar irradiance on a given geographical location requires parametric estimation, that is based on both deterministic and uncertain factors.

III. METHODOLOGY AND ALGORITHMS

As mentioned, solar irradiance values are hindered by numerous factors which make it a challenging task to obtain an estimation for the incident solar insulation. Moreover, the datasets are being affected by the conditions of the surroundings where the physical sensors were installed, especially the underground sensors. The datasets suffered from discontinuities, sensor biases, and energy surges when heavy movable objects pass on the ground above the sensor compartment. Also, the measurements coming from the pyranometer device are affected by the weather conditions, birds and wind blowing. Consequently, the data was manually pre-processed case-by-case to remove the effects of non-parametric causes. Then, the prediction method proceeds with using the state-space estimation concept and Kalman filter algorithms that recognizes only the weather conditions. Later, we intend to develop a more complicated dynamic model to account for other contributing factors.

A. State space estimation

The simplest linear model of state space estimation can be expressed by the following equations:

$$x_k = A_{k-1} x_{k-1} + q_{k-1} \quad (4)$$

$$y_k = H_k x_k + r_k \quad (5)$$

where,

x_k state vector at *k*.

y_k measurement vector at *k*.

q_{k-1} process noise at *k-1* where $q_{k-1} \sim N(0, Q_{k-1})$.

r_k measurement noise at *k* where $r_k \sim N(0, R_k)$.

H_k measurement model matrix at *k*.

x_0 priori distribution where $x_0 \sim N(m_0, P_0)$.

m_0, P_0 initial mean and covariance

N normal distribution function

B. Kalman filters

The Kalman filtering algorithm is an iterative recursive estimation method to predict the new optimal states in linear state space systems considering additive white Gaussian noise. The algorithm is based on utilizing the prior knowledge to estimate the posterior state, then calculate the Kalman gain and the measurements residuals caused by the mismatch error, and finally predict the new state and covariance vectors to be used as an input to the next iteration [3-6].

Basically, the Q_k matrix (process noise covariance matrix) should be discretized using matrix fraction decomposition or the following analytical formula: [6]

$$Q_k = \int_0^{\Delta t_k} \exp\big(F(\Delta t_k - \tau)\big) \times L\, Q_c\, L^T \times \exp\left(F(\Delta t_k - \tau)\right)^T \, d\tau$$

where,

L, F	constant matrices
Q_c	power spectral density matrix
Δt_k	instantaneous time step

IV. NON-LINEAR STATE SPACE ESTIMATION

The nature of most dynamic systems in reality is not linear hence, the linear Kalman Filter cannot be employed to estimate the states of these systems. In case of linear Kalman filter, both system dynamics and measurement process can yield nonlinear output or at least one of them. An extension to Kalman filter is required to deal with such nonlinearity. The solution is the Extended Kalman filter (EKF) for nonlinear state space estimation, which is based on Taylor series approximation of the joint distribution to linearize these systems. In case of severe nonlinear systems, the unscented Kalman filter (UKF) which is based on unscented transformation, is proven to be performing far better than EKF. Other nonlinear state space estimation extensions are developed such as Gauss-Hermite Kalman filter (GHKF) and the third-order symmetric Cubature Kalman filter (CKF). [6-7]

A. Extended Kalman Filter (EKF)

All Kalman filters have two steps: the prediction step, where the next state of the system is predicted given the previous measurements, and the update step, where the current state of the system is estimated given the measurement at that time step. Based on Taylor series approximation, EKF tends to linearize the joint distributions from nonlinear to linear by means of tangential point at each state estimation. Gaussian distribution is assumed all the time in EKF, as follows [6]:

$$x \sim N(m, P) \tag{6}$$

$$y = g(x) \tag{7}$$

where *x* is a normal distribution with *m* as the distribution mean, *P* as the covariance, and *g(.)* is a general nonlinear function of measurements. To solve the distribution of *y* based on *x*, *g* should be Gaussian as well. In this case, *g* is a nonlinear non-Gaussian function so it must be approximated first. The joint distribution of *x* and *y* can be constructed by linear quadratic approximations such as deducing the Jacobian matrix of *g* for each state as follows: [6]

$$G_x(m)_{j,j'} = \frac{\partial g_j(x)}{\partial x_{j'}}\Big|_{x=m} \tag{8}$$

The extended Kalman filter (EKF) extends the scope of the ordinary Kalman filter to nonlinear optimal state estimation problems by forming Gaussian approximation to the joint distribution of the state predictions and measurements using means of Jacobian matrix and Taylor series approximation up to first and second orders, as follows:

$$x_k = f(x_{k-1}, k-1) + q_{k-1} \tag{9}$$

$$y_k = h(x_k, k) + r_k \tag{10}$$

where,

x_k , y_k	state and measurements vectors
q_{k-1} , r_k	process and measurements noise
$f(.)$, $h(.)$	nonlinear functions

B. First Order Extended Kalman Filter

Similar to the ordinary Kalman filter, EKF algorithm consists of two major steps as follows: [8-9]

- Prediction step

$$m_k^- = f(m_{k-1}, k-1) \tag{11}$$

$$P_k^- = F_x(m_{k-1}, k-1)\, P_{k-1}\, {F_x}^T(m_{k-1}, k-1) + Q_{k-1} \tag{12}$$

- Update step

$$V_k = y_k - h(m_k^-, k) \tag{13}$$

$$S_k = H_x(m_k^-, k)\, P_k^-\, {H_x}^T(m_k^-, k) + R_k \tag{14}$$

$$K_k = P_k^-\, {H_x}^T(m_k^-, k)\, S_k^{-1} \tag{15}$$

$$m_k = m_k^- + K_k V_k \tag{16}$$

$$P_k = P_k^- - K_k S_k K_k^T \tag{17}$$

where,

m_k^- , P_k^-	prior mean and covariance
m_k , P_k	posterior mean and covariance
V_k	measurement residual
y_k	measurements vector
S_k	measurement prediction covariance
K_k	filter gain correction coefficient
$F_x(m, k-1)$	Jacobian matrix of function *f*
$H_x(m, k)$	Jacobian matrix of function *h*

The difference between EKF and KF is the replacement of matrices A_k and H_k in Kalman Filter by the Jacobian $F_x(m, k-1)$ and $H_x(m, k)$ in EKF. Thus, predicted mean m_k^-, predicted covariance P_k^- and residual V_k are calculated differently as well.

C. Limitations of EKF

EKF has few disadvantages which somewhat limit its operation as described in [10], that led to the development of the Unscented Kalman Filtering (UKF) to mitigate these limitations. EKF drawbacks can be summarized as follows:

- EKF performs poorly in severe nonlinear models due to the significant approximation.
- Jacobian and Hessian matrices first need to exist in order to perform the transformation.
- Jacobian and Hessian matrices can be very difficult to evaluate in many cases.
- Second-order Kalman Filters require extra computations, which reflects on resources.

An effective way to insert remedies to the EKF output is to complement it with Kalman smoothers [6].

D. Discrete-time Kalman Smoother (RTS)

The Rauch-Tung-Striebel (RTS) smoother was developed by the authors of [9, 11, 12]. RTS can be used for computing the smoothing solution for the state space model given as a distribution. The basic idea here is to use the whole distribution over the whole period *T*, as follows:

$$p(x_k|y_{1:T}) = N(x_k|m_k^s, P_k^s) \tag{18}$$

The mean and covariance m_k^s, P_k^s are calculated using the following formulas:

$$m_{k+1}^- = A_k m_k \tag{19}$$

$$P_{k+1}^- = A_k P_k A_k^T + Q_k \tag{20}$$

$$C_k = P_k A_k^T [P_{k+1}^-]^{-1} \tag{21}$$

$$m_k^s = m_k + C_k[m_{k+1}^s - m_{k+1}^-] \tag{22}$$

$$P_k^s = P_k + C_k[P_{k+1}^s - P_{k+1}^-]C_k^T \tag{23}$$

where,

m_k^s , P_k^s	smoothed mean and covariance
m_k , P_k	mean and covariance
m_{k+1}^- , P_{k+1}^-	predicted mean and covariance
C_k	smoother gain

The difference between Kalman filter and Kalman smoother is the recursive movement of the filter forwards starting from the first-time step *k-1* while the smoother moves backwards starting from the last time step *T*.

E. Extended RTS Kalman Smoother (ERTS)

Similarly, the difference between First-Order EKF smoother and KF smoother is the same as the difference between EKF and KF: the matrices A_k and H_k in Kalman smoother are replaced by the Jacobian $F_x(m, k-1)$ and $H_x(m,k)$ in EKF smoother. Equations of ERTSK smoother become:

$$m_{k+1}^- = f(m_k, k) \tag{24}$$

$$P_{k+1}^- = F_x(m_k, k)\, P_k\, {F_x}^T(m_k, k) + Q_k \tag{25}$$

$$C_k = P_k\, {F_x}^T(m_k, k)\, [P_{k+1}^-]^{-1} \tag{26}$$

$$m_k^s = m_k + C_k[m_{k+1}^s - m_{k+1}^-] \tag{27}$$

$$P_k^s = P_k + C_k[P_{k+1}^s - P_{k+1}^-]C_k^T \tag{28}$$

where,

m_k^s , P_k^s	smoothed mean and covariance
m_k , P_k	mean and covariance
m_{k+1}^- , P_{k+1}^-	predicted mean and covariance
C_k	smoother gain
$F_x(.)$, $H_x(.)$	Jacobian matrices of functions *f* and *h*

V. BUILDING THE SOLAR IRRADIANCE PREDICTION MODEL

The location of Vaasa, Finland and the hourly times of the day have been translated into the following parameters: latitude (ϕ) angle, declination (δ) angle, and hour (*H*) angle.

A. Modelling the Extended Kalman filtering to estimate solar irradiance

As can be concluded from equations of section II, the variables that affect solar irradiance for a fixed location can be reduced to; declination angle (δ) and time of day (*H*), provided that the latitude angle (ϕ) is kept constant. Therefore, the state vector can become $x_k = [\delta_k \quad H_k]^T$
Assuming there are sensor measurements taken every second for both states, then $y_k = [\delta \quad H]$
Vaasa University coordinates are 63°06'11.4"N 21°35'40.1"E, then latitude angle (ϕ) = 63.10322°
It is clear that calculating solar irradiance is a nonlinear state space estimation hence, the extended Kalman filter (EKF) method will be used in this example.

B. Jacobian matrices

EKF uses Jacobian matrices to perform the Gaussian approximation. Assuming that the dynamic function is the same as the measurement function, then the Jacobians in this example become as follows:

$$\mathrm{F_x}(m,k) = \mathrm{H_x}(m,k) = \left[{}^{\partial I}/_{\partial \delta_k} \quad {}^{\partial I}/_{\partial H_k}\right] \tag{29}$$

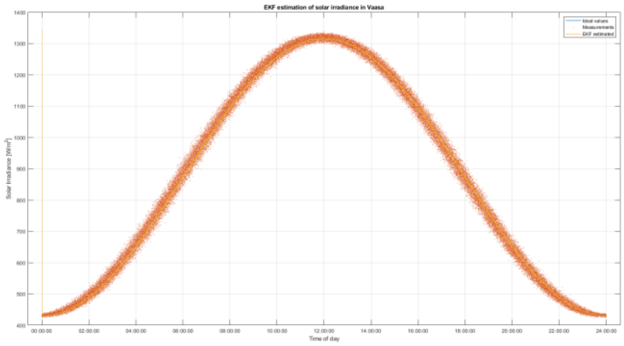
$$= \begin{bmatrix} 1000(-sin\phi.sin\delta - cos\phi.sin\delta.cosH) \\ 1000(0 - cos\phi.cos\delta.sinH) \end{bmatrix}^T \tag{30}$$

C. Initial state vector x_init

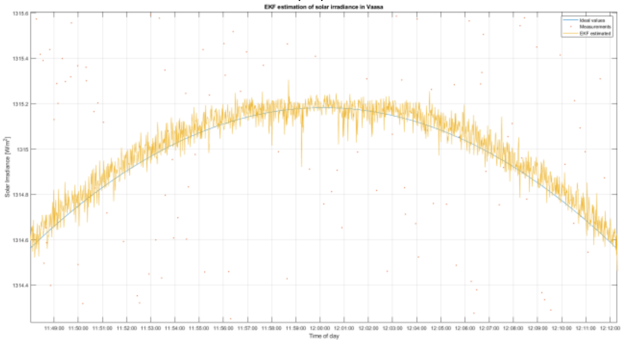
Starting from initial position ($t = 0$) at 00:00, day number is 70 i.e. 11th March 2015, therefore the initial condition vector becomes:

$$x_{init} = [\delta_0 \quad H_0]^T = [-12.10^0 \quad -179.99^0]^T \tag{31}$$

The full MATLAB code for solar irradiance estimation using Extended Kalman filter and ERTS smoother can be found in Appendix I. The results of estimating solar irradiance using Jacobian matrices for a single day are found in Figure 3.

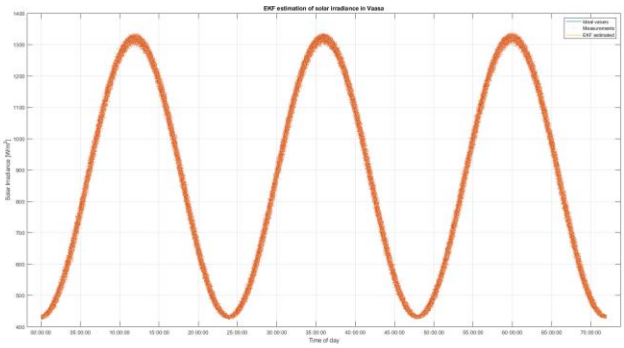


(A)

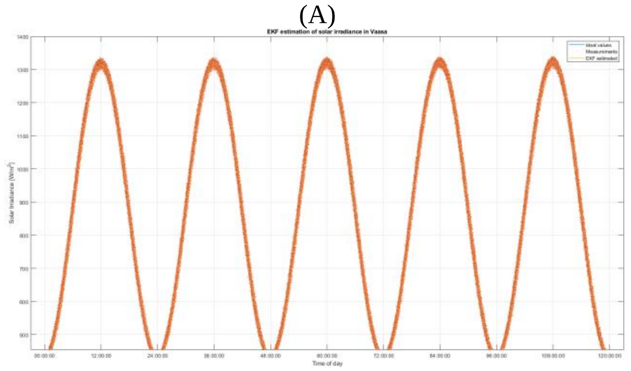


(B)

Figure 3. A) EKF estimations of solar irradiance in Vaasa for the 11th of March 2015. B) A zoomed version of the "A" plot. The blue curve refers to ideal values, red dots refer to measurements, and the yellow curve refers to the filtered values. The results of estimating solar irradiance using Jacobian matrices for n consecutive days starting from the 11th of March 2015 are illustrated in Figure 4.



(A)



(B)

Figure 4. Solar irradiance estimation for several days in March 2015. A) when n = 3 consecutive days, and B) when n = 5 consecutive days.

The mean square error of EKF estimations was:

- `EKF-MSE = 0.0621 watt/m^2`

D. Applying the Extended Kalman smoother (ERTS) algorithm

The ERTS algorithm was used to fine-tune the results of EKF estimates as shown in 5.

Figure 5. The results of ERTS smoother for solar irradiance predictions (11th March 2015).

The Mean Square Error (MSE) of all solar irradiance estimation methods were as follows:

Using EKF: `EKF-MSE = 0.0623 watt/m^2`

Using ERTS: `ERTS-MSE = 0.0614 watt/m^2`

Judging by the given results and plots, clearly the ERTS smoother outperform the original EKF estimations even if with small improvements in the mean square error.

VI. CONCLUSIONS

Renewable energy sources are very crucial for daily life activities especially in remote countries that suffer from solar heat and light deficiencies. As a Nordic country, Finland struggles with providing the necessary energy sources throughout the dark cold winter which prevail for more than six months every year. Energy storage could be a key element for Finland to harvest solar light and heat (warmth) during the long sunny days of summer, which can be used for the small energy burden of summer, and store the excessive for later use. In this article, we experimented the harvesting of incident solar irradiance on the city of Vaasa (Finland) using physical heat sensors buried beneath the earth's surface and a pyranometer device mounted on a building's rooftop. We proposed a simple state-space estimation technique using extended Kalman filter (EKF) and smoother (ERTS) to predict the incident solar irradiation on Vaasa. The results showed an unmatched performance for the given algorithm which yield an acceptable estimation accuracy for the proposed model. For future work, we plan to develop more advanced dynamic model that resembles the solar irradiance on Finland by considering other contributing factors including the extraterrestrial values and atmospheric absorption effects.

REFERENCES

[1] C. Çuhac, A. Mäkiranta, P. Välisuo, E. Hiltunen and M. Elmusrati, "Temperature Measurements on a Solar and Low Enthalpy Geothermal Open-Air Asphalt Surface Platform in a Cold Climate Region," *Energies*, vol. 13, no. 979, 2020.

[2] C. Lynch, M. J. O'Mahony and R. A. Guinee, "A novel 24 kalman filter bank estimator for solar irradiance prediction for PV power generation," *2015 IEEE 42nd Photovoltaic Specialist Conference (PVSC)*, New Orleans, LA, USA, 2015, pp. 1-7.

[3] L. Kleeman, "Understanding and Applying Kalman Filtering," *Proceedings of the Second Workshop on Perceptive Systems*, Curtin University of Technology Perth Western Australia, 1996.

[4] R. E. Kalman, "A New Approach to Linear Filtering and Prediction Problems," *Transactions of the ASME–Journal of Basic Engineering*, vol. 82, pp. 35-45, 1960.

[5] R. Faragher, "Understanding the Basis of the Kalman Filter Via a Simple and Intuitive Derivation," *IEEE Signal Processing Magazine* , pp. 128-132, 2012.

[6] J. Hartikainen, A. Solin and S. Särkkä, Optimal Filtering with Kalman Filters and Smoothers: a Manual for the MATLAB toolbox EKF/UKF, vol. 1.3, Espoo: Aalto University: Department of Biomedical Engineering and Computational Science, 2011.

[7] S. Särkkä, "Recursive Bayesian Inference On Stochastic Differential Equations", Espoo: Helsinki University of Technology Laboratory of Computational Engineering Publications, 2006.

[8] S. Särkkä, A. Vehtari and J. Lampinen, "Rao-Blackwellized particle filter for multiple target tracking," *Information Fusion*, pp. 2-15, 2007.

[9] Y. Bar-Shalom, X.-R. Li and T. Kirubarajan, Estimation with Applications to Tracking and Navigation, Toronota: Wiley-Interscience, 2001.

[10] S. Julier and J. Uhlmann, "Unscented filtering and nonlinear estimation," in *Proceedings of the IEEE*, 2004.

[11] H. E. Rauch, F. Tung and C. T. Striebel, "Maximum likelihood estimates of linear dynamic systems," *AIAA*, vol. 3, no. 8, p. 1445, 1965.

[12] A. Gelb, J. Joseph F. Kasper, J. Raymond A. Nash, C. F. Price and J. Arthuer A. Sutherland, Applied Optimal Estimation, London: MIT Press, 1974.

APPENDIX I

MATLAB code for the nonlinear estimation of solar irradiance in Vaasa, Finland.

```
clc; close all; clear variables;
df_func = @Fx;
dh_func = @Hx;
phi=63.1032222;    % Univaasa latitude
elapsed=86400;      % Elapsed time in
seconds
t=1:elapsed;        % Time iterations
n=2;                 % No. of states
[delta; H]
states=zeros(n,elapsed);                %
Preallocation of state vector
% Initial guesses for delta and Hour
angle H
x_init=[23.45*sind(211.0685-1.1416e-
05);...
        15*((1/3600)-12)];
Y=zeros(2,elapsed);                     %
Preallocation of measurements vector
Yreal=zeros(1,elapsed);                 %
Preallocation states -> real
delta=zeros(1,elapsed);                 %
Preallocation of delta vector
HourAng=zeros(1,elapsed);               %
Preallocation of Hour angle vector
Yrealstates=zeros(2,elapsed);           %
Preallocation of real state vector
for i=1:elapsed
    states(:,i)=[23.45*sind(211.0685-
i*1.1416e-05);...
        15*((1/3600)*i-12)];
    delta(:,i)=states(1,i);
    HourAng(:,i)=states(2,i);

Yrealstates(:,i)=[delta(1,i);HourAng(
1,i)];

Yreal(:,i)=irrad(phi,delta(:,i),HourA
ng(:,i));


delta(:,i)=states(1,i)+gauss_rnd(1,2,
1); % gauss_rnd(mean, var, samples)

HourAng(:,i)=states(2,i)+gauss_rnd(1,
5,1);
 end
Ymstates=[delta;HourAng];               %
Saving measurement vector
Ym=irrad(phi,delta,HourAng);            %
Mapping measurements to values

% Preparing for EKF algorithm
MM   =   zeros(n,elapsed);              %
Preallocation of estimates vector
PP    =    zeros(n,n,elapsed);          %
Preallocation of covariance vector
M=x_init;                      % Initial
state guess
```

```
MM=x_init;
p1=0.1;
p2=0.2;
P=diag([p1  p2]);           %  Initial
covariance guess
r=4;
R=r*eye(n);               % Measurement
noise covariance
q1=0.01;
q2=0.02;
Q=diag([q1 q2]);        % Process noise
covariance
for i=2:elapsed
% Predicition step
M=states(:,i-1);         %  States  via
nonlinear function f
F=diag(Fx(phi,M));       % Calculating
Jacobian F
H=diag(Hx(phi,M));       % Calculating
Jacobian H
% F=eye(n);         % Reducing Jacobian
F to Identity matrix
% H=eye(n);         % Reducing Jacobian
H to Identity matrix
P=F*P*(F)'+Q;              % Predicting
covariance
% Update step
v=Ymstates(:,i)-Yrealstates(:,i);
S=H*P*(H)'+R;
K=P*H'*inv(S);
M=M+K*v;
P=P-K*S*K';
% Saving states & covariance
MM(:,i)    = M;
PP(:,:,i) = P;
end
% Mapping estimated states to values
estimated=zeros(1,elapsed);
for k=1:elapsed

estimated(:,k)=irrad(phi,MM(1,k),MM(2
,k));
end
% Plotting results
figure;
plot(duration(0,0,t),Yreal,
duration(0,0,t),Ym,'+',...

duration(0,0,t),estimated,'linewidth'
,1,'markersize',1)
grid on;
legend('Ideal  values','Measurements',
'EKF estimated')
xlabel('Time of day')
ylabel('Solar Irradiance [W/m^2]')
title('EKF    estimation    of    solar
irradiance in Vaasa')
figure;
plot(duration(0,0,t),Yreal,
duration(0,0,t),estimated)
grid on;
legend('Real values','EKF estimated')
```

```
%  Computing  Mean  square  error  of
estimates
MSE         =         sum((estimated-
Yreal).^2)/elapsed;
fprintf('EKF-MSE                    =
%.4f\n',sqrt(MSE));    %  Result  shown
in Watts/m^2

% Function declaration
%  Irradiance  function(phi,delta,Hour
Angle)
function I=irrad(phi,delta,HourAng)
I=1000.*(sind(phi).*cosd(delta)+cosd(
phi).*cosd(delta).*cosd(HourAng));
end
% Jacboian of nonlinear F equation
function dI = Fx(phi,M)
phi=phi;
delta = M(1,:);
HourAng = M(2,:);
dI   =   [1000*(-sind(phi)*sind(delta)-
cosd(phi)*sind(delta)*cosd(HourAng)),
...
    1000*(0-
cosd(phi)*cosd(delta)*sind(HourAng))]
;
end
% Jacboian of nonlinear H equation
% Assuming same function for F & H
function dI = Hx(phi,M)
phi=phi;
delta = M(1,:);
HourAng = M(2,:);
dI   =   [1000*(-sind(phi)*sind(delta)-
cosd(phi)*sind(delta)*cosd(HourAng)),
...
    1000*(0-
cosd(phi)*cosd(delta)*sind(HourAng))]
;
end
```

MAHMOUD ELSANHOURY is currently pursuing a Ph.D degree in computer science at the University of Vaasa, Finland. He received his M.Sc. (tech) degree in telecommunications engineering from Vaasa University in 2018, and his B.Sc. degree from Alexandria University, Egypt, in 2013. His current research interests cover ubiquitous indoor positioning, fusion-based UWB positioning, satellite positioning (LEO—PNT), and estimation algorithms.

CANER ÇUHAC has been working on wireless sensor networks since 2010. He has received his Doctor of Science degree from University of Vaasa in 2019. His resarch interests consist of wireless sensor applications in military, agricultural and solar energy systems. Currenly he is working in Asyaport Liman A.Ş. as business analyst.

JANNE KOLJONEN is a laboratory engineer and degree programme manager at the University of Vaasa, Finland. He has authored more than 50 scientific articles since 2002. He received a Ph.D. degree in automation technology from the University of Vaasa in 2010.

MOHAMMED ELMUSRATI is a full professor and head of the International program on Sustainable Autonomous Systems (SAS) at the University of Vaasa, Finland. He has published more than 150 papers, books, and book chapters. His research interest includes wireless communications, artificial intelligence, machine learning, biotechnology, big data analysis, stochastic systems, and game theory.